

MF-DFA of the Lithium Market: Evidence of Informational Inefficiency

Abstract

This study assesses the weak-form informational efficiency of the Lithium market, a critical mineral resource essential for the global energy transition, using the Global X Lithium & Battery Tech ETF (LIT) daily returns from 2010 to 2025. We apply the robust Multifractal Detrended Fluctuation Analysis (MF-DFA) to detect long-range correlation and multifractality in this complex, non-stationary time series. Our results reveal a multifractal market structure and deviation from the random walk, indicating greater inefficiency compared to major global stock indices, similar to volatile markets. The generalized Hurst exponent confirms this multifractal structure, exhibiting asymmetric scaling behavior where small price fluctuations show persistent long-range correlation, while large price fluctuations exhibit anti-persistence, reflecting a mean-reverting mechanism. Through analysis of shuffled series, we confirm that the multifractality originates predominantly from temporal dependence (long-range correlation) rather than the heavy-tailed distribution of returns. This deviation from the Efficient Market Hypothesis underscores the need for reforms to enhance transparency and information flow in the Lithium market, fostering optimal capital allocation crucial for sustaining the global energy transition.

JEL codes: C22, C58, G14, G15, Q42, Q55

Keywords: Market efficiency; Lithium market; Critical minerals; MF-DFA; Multifractality; Long-range memory; Energy transition.

1. Introduction

Clean technologies driving the global energy transition are highly mineral-intensive (Kirsten et al., 2020). Among them, lithium stands out as a critical mineral, essential for electric vehicle (EV) batteries and energy storage systems yet exposed to notable supply risks (Carr-Wilson et al., 2024). Global demand is expected to increase 13–51 times by 2040, raising concerns about future supply constraints (IEA, 2021; Ghorbani et al., 2024; Kleijn et al., 2011; Michaux, 2021). These shortages do not arise from geological scarcity but from insufficient investment in extraction and processing capacity (Fleming et al., 2024). Addressing this challenge requires advances in recovery technologies (Vega-Garcia et al., 2023) and, crucially, efficient capital allocation through well-functioning financial markets. Informational efficiency ensures that capital flows optimally to productive uses (Levine, 2005; Fischer and Merton, 1984; Tobin, 1984). Financial efficiency is especially vital in nascent markets like lithium, where capital misallocation can exacerbate future supply constraints.

The conceptual roots of market efficiency were formalized by Fama (1970). According to the weak form of the Efficient Market Hypothesis (EMH), asset returns follow a memoryless stochastic process, implying that past prices hold no predictive power. However, empirical evidence frequently reports long-range dependence (Di Matteo, 2007; Urquhart, 2016; Wei, 2018; Zunino et al., 2008) and multifractality in financial time series, suggesting that real markets often deviate from full informational efficiency. Despite the extensive application of these tests across various financial assets, the informational efficiency of critical minerals such as lithium remains largely unexplored, representing a significant gap in the literature.

To address this significant gap, this study applies the Multifractal Detrended Fluctuation Analysis (MF-DFA), an advanced econophysics technique that is well-suited for capturing the nonlinear, chaotic, and fractal dynamics characteristic of non-stationary financial time series (Mensi et al., 2018; Rizvi et al., 2014; Shrestha et al., 2023; Suárez-García and Gómez-Ullate, 2014). MF-DFA's capability to differentiate scaling behavior between small and large fluctuations allows for a nuanced assessment of long-term memory and heterogeneity, providing direct evidence for or against the weak-form EMH (Acikgoz, 2025).

Since physical lithium is not directly tradable, the Global X Lithium & Battery Tech ETF (LIT) serves as a representative proxy, providing a long, liquid time series that captures the market dynamics of this critical asset. Fractional Gaussian Noise (fGn), shuffled series, and major stock indices (S&P 500 and Nasdaq) are employed as benchmarks to identify the source of multifractality and gauge efficiency relative to mature markets.

This work contributes in three main ways: (i) it provides the first comprehensive assessment of informational efficiency and long-term memory in the lithium market; (ii) it employs MF-DFA, a robust method capturing multifractal, nonstationary dynamics and differentiating the scaling of small and large price fluctuations; and (iii) it offers new empirical evidence that the lithium market exhibits weak inefficiency, marked by persistent small fluctuations and mean-reverting large ones, driven primarily by temporal dependence rather than fat-tailed distributions. These findings underscore the importance of enhancing transparency and information flow to foster efficient capital allocation in the expanding lithium economy.

This paper is organized as follows. Section 2 reviews the relevant literature and provides the theoretical foundation for the Multifractal Detrended Fluctuation Analysis (MF-DFA). Section 3 details the data used in this study and describes the MF-DFA methodology and the metrics employed. Section 4 discusses the empirical results, including the analysis of multifractality, efficiency metrics, and the source of long-range correlation. Finally, Section 5 presents the conclusions and policy implications.

2. Literature Review

The assessment of financial market efficiency remains a crucial area of inquiry for academics, investors, and policymakers, primarily because efficient resource allocation relies on well-functioning markets. The conventional framework, articulated by the EMH in its weak form, posits that asset prices reflect all historical price information instantly, implying that returns follow a random walk process. Consequently, in a weak-form efficient market, predicting prices is deemed impossible, and investors cannot earn abnormal returns through historical price analysis.

However, this traditional viewpoint has faced increasing challenge due to empirical evidence revealing stylized facts in financial time series that contradict the EMH, including volatility clustering, non-linearity, and long-range dependence (Miloš et al., 2020; Shahzad et al., 2017). The presence of multifractality and long-memory behavior directly supports the notion of market inefficiency, which has spurred the adoption of fractal models and econophysics methods to accurately capture these complex dynamics.

To reliably analyze complex and nonstationary financial time series, the Multifractal Detrended Fluctuation Analysis (MF-DFA) method, developed by Kantelhardt et al. (2002), has become the

standard approach. MF-DFA is a generalization of the standard Detrended Fluctuation Analysis (DFA). It is widely recognized as a robust tool for assessing market efficiency under complex and nonlinear conditions (Kantelhardt et al., 2002; Mensi et al., 2018; Rizvi et al., 2014; Shahzad et al., 2017). The core strength of MF-DFA lies in its ability to quantify multiple scaling exponents within a time series, distinguishing it from monofractal approaches where a single scaling exponent may lead to spurious conclusions. This capability allows researchers to assess long-range dependence, efficiency, and persistence by examining how the generalized Hurst exponent, $h(q)$, varies across different magnitudes of price fluctuations. This allows MF-DFA to capture the inherent heterogeneity (multifractality) of the underlying dynamics. For a financial series that follows a random walk (indicating efficiency), $h(q)$ approx 0.5. Values of $h(q)$ between 0.5 and 1 indicate persistence or long-term memory, while values between 0 and 0.5 suggest anti-persistence (mean-reverting behavior). The generalized Hurst exponent $h(q)$ is sensitive to the value of q : negative values emphasize small fluctuations, while positive values highlight large fluctuations. Furthermore, MF-DFA quantifies the complexity and heterogeneity of the market through the multifractal spectrum $f(\alpha)$ and its width $\Delta\alpha$. A smaller spectral width suggests lower heterogeneity, higher market efficiency, and lower market risk. To quantify market inefficiency, several studies adopt the Market Deficiency Measure (MDM), which calculates the deviation of the generalized Hurst exponents (for small and large fluctuations) from the efficient market baseline (0.5) (Al-Yahyaee et al., 2018; Bouoiyour et al., 2018; Tiwari et al., 2019; Wang et al., 2009; Wan et al., 2025).

MF-DFA has been widely applied across diverse asset classes to analyze multifractality and weak-form efficiency. Research frequently examines stock markets, finding widespread evidence of multifractality and time-varying efficiency. Comparisons between developed and emerging stock markets generally show that developed markets tend to exhibit higher efficiency (Mensi et al., 2018; Rizvi et al., 2014; Bouoiyour et al., 2018; Miloš et al., 2020; Tiwari et al., 2019).

Other non-traditional assets have also been analyzed applying MF-DFA: A comparative analysis of the Bitcoin market versus gold, stock, and currency markets (Al-Yahyaee et al., 2018) found that Bitcoin exhibited stronger multifractality and was therefore more inefficient than the other markets. Similarly, the long-term memory properties of crude oil markets have been extensively examined using MF-DFA (Gu et al., 2010; Zhuang et al., 2015; Ma et al., 2014). MF-DFA has also been used to assess the efficiency of Credit Default Swap (CDS) and equity markets at the industry level, finding that CDS markets were generally more inefficient than their equity counterparts (Shahzad et al., 2017).

Beyond quantifying efficiency through $h(q)$ and $\Delta\alpha$, a key finding provided by MF-DFA analysis is the ability to determine the origin of multifractality in financial returns. Multifractality is generally attributed to two main factors: different long-range correlations for small and large fluctuations (temporal dependence), or fat-tailed probability distributions of variations. By comparing the original time series with randomized (shuffled), researchers can isolate the influence of temporal correlations versus distribution properties (Cao et al., 2013; Green et al., 2014; Thompson and Wilson, 2016; Wan et al., 2025). For instance, if shuffling the series significantly reduces fractality, the source is temporal dependence (long-range memory). Despite the versatility and widespread application of MF-DFA to conventional stock indices, emerging markets, cryptocurrencies, and energy commodities, the informational efficiency and long-term memory of critical minerals markets essential for the global energy transition remain largely unexplored. This study addresses this significant gap by providing the first comprehensive application of the MF-DFA methodology to the Lithium market.

3. Data and Methodology

LIT ETF tracks the Solactive Global Lithium Index, featuring major firms in lithium extraction and lithium-ion battery production (Solactive, 2024). According to Corzo et al. (2022), the collective behavior of companies within an industry effectively reflects the dynamics of the underlying asset. Unlike illiquid futures, LIT ETF offers a reliable way to capture the dynamics of this critical mineral. We use closing prices from Bloomberg to compute daily logarithmic returns $r_t = \log(P_t) - \log(P_{t-1})$ from the LIT ETF's inception on July 26, 2010, to September 30, 2025, resulting in 3,913 observations. Cumulative returns are presented in Figure 1, and summary statistics are provided in Table 1, with the S&P 500 and Nasdaq (sourced from FactSet) serving as reference indices.

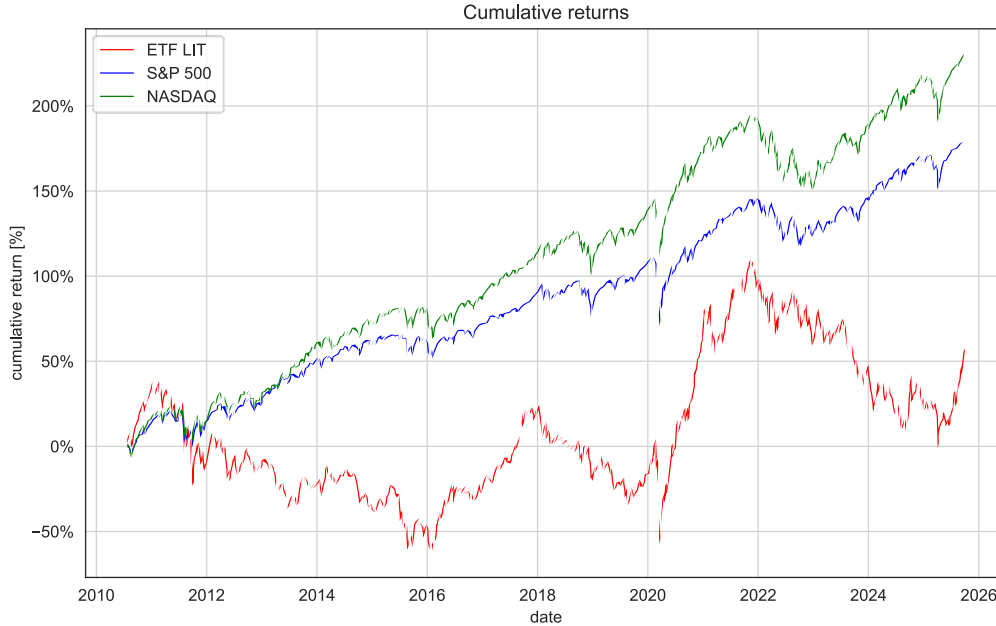


Figure 1: Cumulative returns of the LIT ETF and reference stock indices.

Table 1: Descriptive statistics of returns for the LIT ETF and reference stock indices. Jarque-Bera tests reject the normality of returns at the 1% level, with a p -value of 0 in all cases.

	Num. Observ.	Mean %	Std %	Min %	Max %	Skew	Kurtosis
ETF LIT	3963	0.0144	1.745	-14.60	13.88	-0.18	5.51
S&P 500 Index	3963	0.0455	1.071	-12.77	9.09	-0.64	15.00
NASDAQ Index	3963	0.0581	1.270	-13.15	11.48	-0.46	9.10

Although the mean return is close to zero, the return distribution departs from normality, as indicated by mild negative skewness and pronounced excess kurtosis. The slightly negative skewness points to a limited asymmetry toward negative returns, while the high kurtosis signals the presence of fat tails and an elevated incidence of extreme observations. From a market efficiency perspective, such non-Gaussian features indicate the potential presence of nonlinear dynamics and heterogeneous volatility regimes. In particular, the substantial excess kurtosis suggests that extreme fluctuations may play a non-negligible role in the temporal structure of returns, motivating the use of multifractal methods such as MF-DFA to investigate whether scaling properties vary across fluctuation magnitudes.

Methodologically, MF-DFA offers clear advantages by removing local trends and noise through polynomial detrending, preventing spurious long-range dependence in nonstationary data (Chen et al., 2025; David et al., 2020; Kantelhardt et al., 2002). The method is grounded in econophysics and allows multiple scale exponents to be quantified to avoid spurious conclusions that arise with monofractal methods.

We follow Kantelhardt et al. (2002) for analyzing a non-stationary time series of cumulative log-returns (X_t)

$$Y_i = \sum_{k=1}^i (X_k - \mu_X), \quad \text{for } i = 1, 2, \dots, N$$

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^s [Y_{(v-1)s+i} - y_{(v-1)s+i}]^2$$

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(v, s)]^{q/2} \right\}^{1/q}$$

To establish the generalized fluctuation function $F_q(s)$, we compute a profile Y_t of the input series X_t and assess the fluctuations $F^2(v, s)$ between this profile and a polynomial fit y_t that represents the trend across $2N_s$ non-overlapping segments of length s . In this study, y_t uses linear detrending.

$F_q(s)$ depends on the segment size s and power q . For $q=2$, it simplifies to monofractal DFA. The function $F_q(s)$ follows the exponential law:

$$F_q(s) \sim s^{h(q)}, \quad h(q) \sim \frac{\log F_q(s)}{\log s}$$

The generalized Hurst exponent $h(q)$ measures how local fluctuations grow with scale s , with q distinguishing between small ($q < 0$) and large ($q > 0$) fluctuations. Asset prices following a random walk, exhibit $h(q) \approx 0.5$; values between 0 and 0.5 indicate negative correlation (antipersistence), whereas values ranging from 0.5 to 1 represent positive correlation (persistence). A time series is considered multifractal if $h(q)$ changes with q ; otherwise, a stable $h(q)$ indicates monofractality (Mandelbrot, 1989).

MF-DFA estimates the multifractal spectrum $D(\alpha)$ as a function of the Hölder exponent α . The Hölder exponent represents singularity strength, whereas the complexity at each α level, $D(\alpha)$, is assessed via a Legendre transform of the Rényi exponent $\tau(q)$:

$$\tau(q) = qh(q) - 1, \quad \alpha = \tau'(q) = h(q) + qh'(q), \quad D(\alpha) = q\alpha - \tau(q)$$

The peak of the spectrum α^* indicates the prevailing scaling behavior. Efficient series typically display a peak at $\alpha^* \approx 0.5$ and a narrower spectrum $\Delta\alpha = \max(\alpha) - \min(\alpha)$, implying reduced heterogeneity, greater market efficiency, and diminished risk (Zunino et al., 2008). Following Wang et al. (2009), we also compute the Market Deficiency Metric (MDM)—the deviation of minimum and maximum Hurst exponents from the efficient market baseline (0.5)—to quantify market inefficiency.

The value of MDM is zero if the market is efficient, i.e., if all fluctuations (small and large) follow a random walk process. Conversely, a high value indicates a less efficient market.

$$\text{MDM} = \frac{|h(q_{\min}) - 0.5| + |h(q_{\max}) - 0.5|}{2}$$

In this study, we use $q_{\min}=-4$ and $q_{\max}=+4$ to calculate the deviation in small and large fluctuations, respectively, as is done in recent works (Mensi et al., 2021; Wan et al., 2025)

Because the multifractal spectrum lacks a universal threshold, interpretation requires comparison (Mandelbrot, 1989; Thompson and Wilson, 2016). We therefore use Fractional Gaussian Noise (fGn) and shuffled series as benchmarks. The fGn spectrum, generated with $H = 0.5$ and averaged over 10,000 simulations, serves as a monofractal baseline to gauge data complexity (Mandelbrot and Van Ness, 1968; Taqqu et al., 1995). Shuffling the series helps identify the source of multifractality by removing long-range correlations, thereby narrowing the spectrum toward 0.5 (Kantelhardt et al., 2001). A pronounced reduction in the spectral width after shuffling indicates that multifractality originates from temporal dependence (long-range memory); otherwise, it stems from broad probability distributions such as fat tails (Green et al., 2014; Thompson and Wilson, 2016).

3. Results

Following the guidelines of Thompson and Wilson (2016) and employing the Python MF-DFA library by Gorjão et al. (2022), our analysis reveals that the lithium market exhibits a complex dynamic structure, providing key insights into its weak-form efficiency. The initial results confirm significant multifractal patterns, as evidenced by the generalized Hurst exponent, $h(q)$, which varies significantly with the weighting order q (Figure 2). If $h(q)$ were constant, the series would be monofractal. The non-constancy of $h(q)$ across q is direct evidence that the market is intrinsically multifractal, thus rejecting the random walk hypothesis and the weak-form EMH. This reveals distinct price correlations across different magnitudes of fluctuation.

For $q < 0$, the generalized Hurst exponent $h(q)$ describes the scaling behavior of small fluctuations. Specifically, small fluctuations exhibit persistent behavior ($0.5 < h(q) < 1$), reflecting a trend-reinforcing mechanism (small price movements tend to be followed by further movements in the same direction). $H(q)$ values greater than 0.5 in the case of small fluctuations indicate that the series has the property of long-range memory, which translates into persistence. Conversely, for $q > 0$, the scaling exponents $h(q)$ describe the scaling behavior of large fluctuations. Here, large price fluctuations exhibit anti-persistent behavior ($0 < h(q) < 0.5$), implying that an increase (decrease) in price is more likely to be followed by a decrease (increase), indicating a process of reversion to the mean (Mensi et al., 2017). The coexistence of persistence in small fluctuations and anti-persistence or mean reversion in large fluctuations is a key behavioral asymmetry characteristic detected in the Lithium market, similar to that found in other volatile markets such as Bitcoin.

The dominant scaling exponent (α^*), representing the peak of the singularity spectrum, captures the market's prevailing correlation structure and reflects the most frequent scaling behavior (singularity strength) of the time series. This exponent serves as a key indicator of informational efficiency. In the lithium market, the measured α^* value is 0.555, which reveals a persistent scaling behavior, clearly deviating from the weak-form efficient market benchmark of $\alpha^* \approx 0.5$. This observed deviation explicitly rejects the random walk assumption and indicates a persistent temporal correlation structure. This persistence can be linked to major technological advances, speculative cycles, and

macroeconomic developments in the sector. Notable factors include the early advancements in lithium-ion batteries and the increasing adoption of EV that generated optimism from 2013 to 2015. Additionally, the speculative boom from 2017 to 2020, driven by EV expansion, subsidies, and the decreasing costs of renewable energy, led to heightened institutional investment and herding behavior. This phase was ultimately followed by concerns over oversupply, resulting in a market correction. In contrast, mature markets, such as the S&P 500 and Nasdaq, exhibit a closer mean-reverting behavior with α^* values of 0.439 and 0.437, respectively (Figure 2 and Table 2). The finding of $\alpha^* > 0.5$ for the Lithium ETF is consistent with results from other emerging or critical mineral markets, where positive autocorrelation (persistence) is prevalent (Mensi et al., 2018).

The complexity of the lithium market is quantified through the spectral width ($\Delta\alpha$), a key metric used to interpret market heterogeneity and risk (Table 2). The lithium market demonstrates the widest spectrum ($\Delta\alpha = 0.732$), clearly indicating a greater degree of multifractality and inefficiency. As the spectral width $\Delta\alpha$ quantifies the range of correlation behavior, this wide spectrum indicates high heterogeneity and greater market risk compared to mature stock indices. This is markedly higher than the theoretical efficient benchmark of Fractional Gaussian Noise (fGn) ($\Delta\alpha = 0.129$) as well as the randomly shuffled series ($\Delta\alpha = 0.277$), and stock market references (as evidenced in Figure 2 and 3). Furthermore, the MDM reinforces this finding, showing greater inefficiency for the Lithium market compared to any other benchmark (MDM = 0.181).

To verify the source of multifractality, we compare the results from the original LIT ETF series with those from randomly shuffled series. The randomized series preserves the statistical distribution (e.g., fat tails) but eliminates long-range correlations. This is evident as shuffling significantly reduces fractal complexity, with the spectral width dropping from 0.732 to 0.277 (see Table 2). A pronounced reduction in fractality after randomization indicates that multifractality originates from temporal dependence (long-range correlation). If multifractality originated primarily from distribution properties (fat tails), the spectrum of the randomized series would not have narrowed dramatically (Cao et al., 2013). Thus, the observed multifractality is not mainly due to broad probability density functions, but suggests that price formation is influenced by investor behavior and market dynamics, indicating that past price movements have predictive power.

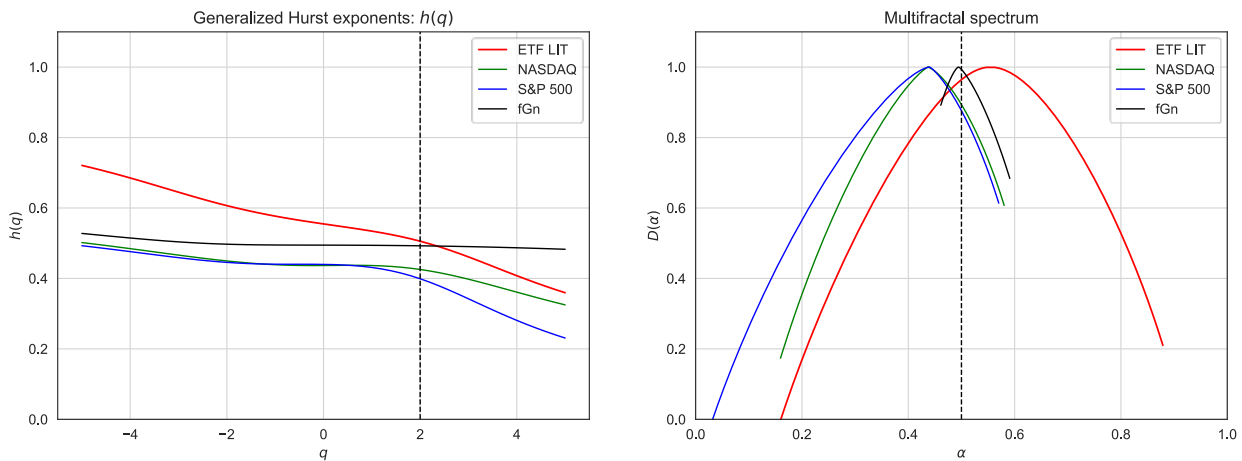


Figure 2: Multifractal DFA Generalized Hurst Exponent (left) and Singularity Spectrum (right) for Lithium, stock indices, and fGn series obtained through 10,000 simulations.

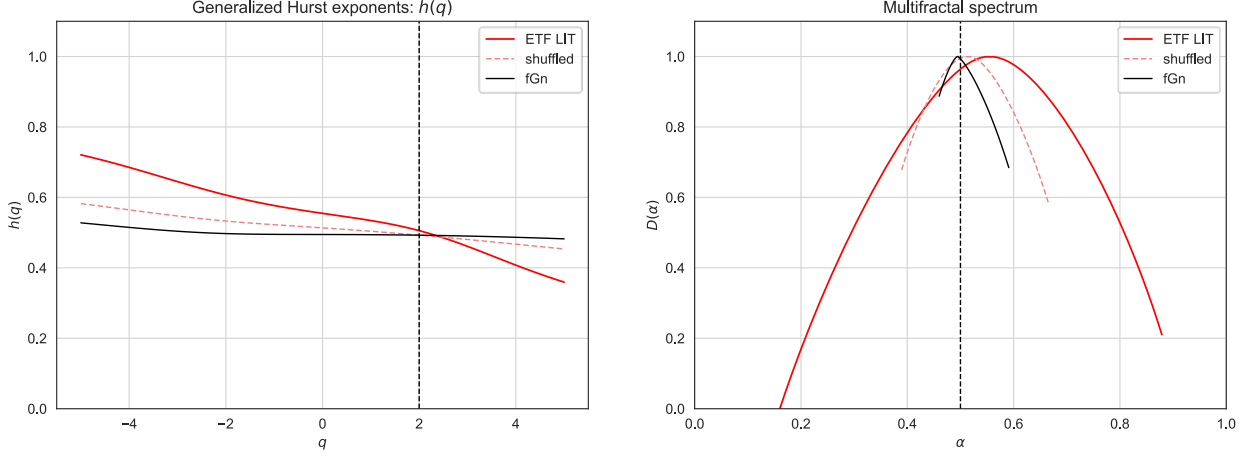


Figure 3: Multifractal DFA Generalized Hurst Exponent (left) and Singularity Spectrum (right) for Lithium, Lithium shuffled, and fGn series obtained through 10,000 simulations.

Table 2: Multifractal Metrics including fGn, shuffled series, and stock indices. $\Delta\alpha$ represents the spectral width, α^* the dominant Hölder coefficient, and MDM the Market Deficiency Metric.

	$\Delta\alpha$	α^*	MDM
fGn	0.129	0.495	0.022
LIT ETF	0.732	0.555	0.181
<i>shuffled</i>	0.277	0.513	0.065
S&P 500	0.554	0.439	0.138
NASDAQ	0.421	0.437	0.088

4. Conclusions

This paper assesses the weak-form efficiency of the lithium market using the LIT ETF and the robust MF-DFA methodology. Our results confirm the strong multifractality of the lithium market, evidenced by the non-constant Generalized Hurst Exponent $h(q)$, which decisively rejects the weak-form EMH.

This multifractality is characterized by persistent correlations and long-range memory. Compared to major stock indices, the lithium market shows much higher multifractality and efficiency deviation, similar to the levels of inefficiency and multifractality found in highly volatile, non-traditional markets, such as Bitcoin.

The coexistence of persistence in small fluctuations ($q < 0$) and anti-persistence in large fluctuations ($q > 0$) is a key behavioral characteristic. This implies that past price movements have predictive power, creating potential arbitrage opportunities for informed investors.

The market's dynamics exhibit high heterogeneity, quantified by the wide spectral width and a dominant persistent correlation structure. This inefficiency is fundamentally structural, as confirmed by the comparison with shuffled data, which shows that long-range temporal correlations are the

dominant source of multifractality. This indicates that the multifractal structure is predominantly structural, rather than being induced merely by the distribution properties of returns (fat tails).

Overall, the lithium market remains weakly inefficient, revealing limited financial maturity and potential arbitrage opportunities for informed investors, since long-range dependence implies return predictability. We recommend that policymakers prioritize structural reforms to enhance transparency liquidity, and the flow of information, which are crucial for improving financial maturity and ensuring efficient capital allocation into extraction and processing necessary for the global energy transition.

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5. References

- Acikgoz, T. 2025. Globalization and stock market efficiency. *Finance Res. Lett.*, 107955. <https://doi.org/10.1016/j.frl.2025.107955>
- Al-Yahyaee, K. H., Mensi, W., Yoon, S. M. 2018. Efficiency, multifractality, and the long-memory property of the Bitcoin market: A comparative analysis with stock, currency, and gold markets. *Finance Res. Lett.*, 27, 228–234. <https://doi.org/10.1016/j.frl.2018.03.017>
- Bouoiyour, J., Selmi, R., Wohar, M. E. 2018. Are Islamic stock markets efficient? A multifractal detrended fluctuation analysis. *Finance Res. Lett.*, 26, 100–105. <https://doi.org/10.1016/j.frl.2017.12.008>
- Cao, G., Cao, J., Xu, L. 2013. Asymmetric multifractal scaling behavior in the Chinese stock market: Based on asymmetric MF-DFA. *Physica A*, 392, 797–807. <https://doi.org/10.1016/j.physa.2012.10.042>.
- Carr-Wilson, S., Pattanayak, S. K., Weinthal, E. 2024. Critical mineral mining in the energy transition: A systematic review of environmental, social, and governance risks and opportunities. *Energy Res. Soc. Sci.*, 116, 103672. <https://doi.org/10.1016/j.erss.2024.103672>
- Chen, F., Yin, S., Zhang, J., Sha, Y., Ji, H. 2025. The interplay between multifractal characteristics and seasonal fluctuations within the LNG spot freight market: insights, forecasting, and trading strategies. *Nonlinear Dyn.*, 113(3), 2915–2939. <https://doi.org/10.1007/s11071-024-10343-1>
- Corzo, T., Martin-Bujack, K., Portela, J., Sáenz-Diez, R. 2022. Early market efficiency testing among hydrogen players. *Int. Rev. Econ. Financ.*, 82, 723–742. <https://doi.org/10.1016/j.iref.2022.08.011>

- David, S. A., Inácio Jr, C. M. C., Quintino, D. D., Machado, J. A. T. 2020. Measuring the Brazilian ethanol and gasoline market efficiency using DFA-Hurst and fractal dimension. *Energy Econ.*, 85, 104614. <https://doi.org/10.1016/j.eneco.2019.104614>
- Di Matteo, T. 2007. Multi-scaling in finance. *Quant. Finance*, 7(1), 21–36. <https://doi.org/10.1080/14697680600969727>
- Fama, E. F. 1970. Efficient capital markets: A review of theory and empirical work. *J. Finance*, 25(2), 383–417. <https://doi.org/10.2307/2325486>
- Fischer, S., Merton, R.C. 1984: Macroeconomics and finance: The role of the stock market. In *Carnegie-Rochester conference series on public policy* (Vol. 21, pp. 57-108). North-Holland.
- Fleming, M., Kannan, S. G., Eggert, R. 2024. Long-run availability of mineral resources: The dynamic case of lithium. *Resour. Policy*, 97, 105226. <https://doi.org/10.1016/j.resourpol.2024.105226>
- Ghorbani, Y., Zhang, S. E., Bourdeau, J. E., Chipangamate, N. S., Rose, D. H., Valodia, I., Nwaila, G. T. 2024. The strategic role of lithium in the green energy transition: Towards an OPEC-style framework for green energy-mineral exporting countries (GEMEC). *Resour. Policy*, 90, 104737. <https://doi.org/10.1016/j.resourpol.2024.104737>
- Gorjão, L. R., Hassan, G., Kurths, J., Witthaut, D. 2022. MFDFA: Efficient multifractal detrended fluctuation analysis in python. *Comput. Phys. Commun.*, 273, 108254. <https://doi.org/10.1016/j.cpc.2021.108254>
- Green, E., Hanan, W., Heffernan, D. 2014. The origins of multifractality in financial time series and the effect of extreme events. *Eur. Phys. J. B*, 87, 1–9. <https://doi.org/10.1140/epjb/e2014-50064-x>
- Gu, R., Chen, H., Wang, Y. 2010. Multifractal analysis on international crude oil markets based on the multifractal detrended fluctuation analysis. *Physica A*, 389, 2805–2815. <https://doi.org/10.1016/j.physa.2010.03.003>
- IEA 2021. The Role of Critical Minerals in Clean Energy Transitions. International Energy Association, Paris. <https://www.iea.org/reports/the-role-of-critical-minerals-in-clean-energy-transitions>
- Kantelhardt, J. W., Koscielny-Bunde, E., Rego, H. H., Havlin, S., Bunde, A. 2001. Detecting long-range correlations with detrended fluctuation analysis. *Physica A*, 295(3-4), 441–454. [https://doi.org/10.1016/S0378-4371\(01\)00144-3](https://doi.org/10.1016/S0378-4371(01)00144-3)
- Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., Stanley, H. E. 2002. Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A*, 316(1-4), 87-114. [https://doi.org/10.1016/S0378-4371\(02\)01383-3](https://doi.org/10.1016/S0378-4371(02)01383-3)
- Kirsten, Hund, Daniel La, Porta, Fabregas, Thao P., Tim, Laing, Drexhage, 2020. Minerals for climate action: The mineral intensity of the clean energy transition. Technical report, World Bank, 2020. <https://documents.worldbank.org/en/publication/documents-reports/documentdetail/099052423172525564>
- Kleijn, R., Van der Voet, E., Kramer, G. J., Van Oers, L., Van der Giesen, C. 2011. Metal requirements of low-carbon power generation. *Energy*, 36(9), 5640-5648. <https://doi.org/10.1016/j.energy.2011.07.003>
- Levine, R. 2005. Finance and growth: Theory and evidence. *Handbook of Economic Growth*, 1, 865-934. [https://doi.org/10.1016/S1574-0684\(05\)01012-9](https://doi.org/10.1016/S1574-0684(05)01012-9)

- Ma, F., Zhang, Q., Chen, Z., Wei, C. 2014. Multifractal detrended cross-correlation analysis of the oil-dependent economies: Evidence from the West Texas intermediate crude oil and the GCC stock markets. *Physica A*, 154–166. <https://doi.org/10.1016/j.physa.2014.05.023>
- Mandelbrot, B. 1989. Multifractal measures, especially for the geophysicist. *Pure Appl. Geophys.*, 5–42. <https://doi.org/10.1007/BF00874478>
- Mandelbrot, B., Van Ness, J. W. 1968. Fractional Brownian motions, fractional noises and applications. *SIAM Rev.*, 10(4), 422–437. <https://doi.org/10.1137/1010093>
- Mensi, W., Hamdi, A., Yoon, S. M. 2018. Modelling multifractality and efficiency of GCC stock markets using the MF-DFA approach: A comparative analysis of global, regional and Islamic markets. *Physica A*, 503, 1107–1116. <https://doi.org/10.1016/j.physa.2018.08.130>
- Mensi, W., Tiwari, A. K., Yoon, S. M. 2017. Global financial crisis and weak-form efficiency of Islamic sectoral stock markets: An MF-DFA analysis. *Physica A*, 471, 135–146. <https://doi.org/10.1016/j.physa.2016.12.034>
- Mensi, W., Vo, X. V., Kang, S. H. 2021. Upside-downside multifractality and efficiency of green bonds: the roles of global factors and COVID-19. *Finance Res. Lett.*, 43, 101995. <https://doi.org/10.1016/j.frl.2021.101995>
- Michaux, S. P. 2021. The mining of minerals and the limits to growth. Geological Survey of Finland: Espoo, Finland. Report 16/2021
- Miloš, L. R., Hațiegan, C., Miloș, M. C., Barna, F. M., Boțoc, C. 2020. Multifractal Detrended Fluctuation Analysis (MF-DFA) of Stock Market Indexes. Empirical Evidence from Seven Central and Eastern European Markets. *Sustainability*, 12, 535. <https://doi.org/10.3390/su12020535>
- Rizvi, S. A. R., Dewandaru, G., Bacha, O. I., Masih, M. 2014. An analysis of stock market efficiency: Developed vs Islamic stock markets using MF-DFA. *Physica A*, 407, 86–99. <https://doi.org/10.1016/j.physa.2014.03.091>
- Shahzad, S. J. H., Nor, S. M., Mensi, W., Kumar, R. R. 2017. Examining the efficiency and interdependence of US credit and stock markets through MF-DFA and MF-DXA approaches. *Physica A*, 471, 351–363. <https://doi.org/10.1016/j.physa.2016.12.037>
- Shrestha, K., Naysary, B., Philip, S. S. S. 2023. Fintech market efficiency: A multifractal detrended fluctuation analysis. *Financ. Res. Lett.*, 54, 103775. <https://doi.org/10.1016/j.frl.2023.103775>
- Solactive German Index Engineering. (2024). FACTSHEET Solactive Global Lithium Index. Frankfurt. <https://www.solactive.com/Indices/?index=DE000A1EY8J4>
- Suárez-García, P., Gómez-Ullate, D. 2014. Multifractality and long memory of a financial index. *Physica A*, 394, 226–234. <https://doi.org/10.1016/j.physa.2013.09.038>
- Taqqu, M. S., Teverovsky, V., Willinger, W. 1995. Estimators for long-range dependence: An empirical study. *Fractals*, 3(04), 785–798. <https://doi.org/10.1142/S0218348X95000692>
- Thompson, J. R., Wilson, J. R. 2016. Multifractal detrended fluctuation analysis: Practical applications to financial time series. *Math. Comput. Simul.*, 126, 63–88. <https://doi.org/10.1016/j.matcom.2016.03.003>
- Tiwari, A. K., Aye, G. C., Gupta, R. 2019. Stock market efficiency analysis using long spans of data: A multifractal detrended fluctuation approach. *Finance Res. Lett.*, 28, 398–411. <https://doi.org/10.1016/j.frl.2018.06.012>
- Tobin, J. 1984. On the efficiency of the financial-system. *Lloyds Bank Annual Review*, (153), 1–15.

- Urquhart, A. 2016. The inefficiency of Bitcoin. *Econ. Lett.*, 148, 80-82.
<https://doi.org/10.1016/j.econlet.2016.09.019>
- Vega-Garcia, L., Ho, Y. C., Myo Thant, M. M., Han, D. S., Lim, J. W. 2023. Lithium in a sustainable circular economy: A comprehensive review. *Processes*, 11(2), 418.
<https://doi.org/10.3390/pr11020418>
- Wei, W. C. 2018. Liquidity and market efficiency in cryptocurrencies. *Econ. Lett.*, 168, 21-24.
<https://doi.org/10.1016/j.econlet.2018.04.003>
- Wan, P., Mujtaba, G., Ashfaq, S., Liangrong, S., Nasir, R. M. 2025. Are rare earth stocks efficient? Novel insights using asymmetric MF-DFA. *Financ. Innov.*, 11:49. <https://doi.org/10.1186/s40854-024-00744-x>
- Wang, Y., Liu, L., Gu, R. 2009. Analysis of efficiency for Shenzhen stock market based on multifractal detrended fluctuation analysis. *Int. Rev. Financ. Anal.*, 18(5), 271-276
<https://doi.org/10.1016/j.irfa.2009.09.005>
- Zhuang, X., Wei, Y., Ma, F. 2015. Multifractality, efficiency analysis of Chinese stock market and its cross-correlation with WTI crude oil price. *Physica A*, 430, 101–113.
<https://doi.org/10.1016/j.physa.2015.02.085>
- Zunino, L., Tabak, B. M., Figliola, A., Pérez, D. G., Garavaglia, M., Rosso, O. A. 2008. A multifractal approach for stock market inefficiency. *Physica A*, 387(26), 6558–6566.
<https://doi.org/10.1016/j.physa.2008.08.028>